CHAPTER FIVE

CAPITALIZATION/DISCOUNT RATES

"Money doesn't always bring happiness
People with ten million dollars are no happier
than people with nine million dollars."
Hobart Brown (d. 2001)
Artist, Sculptor

I. OVERVIEW

Calculation of an appropriate capitalization/discount rate is one of the most difficult, and critical, steps in valuing a business or business interest. It is also a frequently contested area, since there is no single method or formula to arrive at the discount or capitalization rate. The discussion presented in this chapter is introductory—an overview of the concept and some of the tools most often used to compute the rates. The subject matter is so vast that whole courses on the topic of capitalizing and discounting are taught throughout the industry.

An equity interest in a closely held business should be considered an investment on which the holder expects a return. Investors will hold a security only if its expected return is high enough to compensate for any risk.

Within the context of business valuations, the capitalization or discount rate is the "yield rate" on the business investment. The yield rate is comprised of two main elements.

1. Safe (or reasonable) rate of return on secure investments (valuation analysts sometimes like to match the safe rate to the holding period of the investment).
2. An additional return (premium) that compensates the investor for the relative degree of risk, in excess of the safe rate, inherent in the investment.

A. RISK ADJUSTMENT FACTORS

From a risk adjustment standpoint, there are three main categories of factors that may influence the capitalization or discount rate. Specific factors affecting risk are listed for each category. The three categories (and examples of factors) are:

1. External Factors

a) Expectations of the general economy
b) Existing conditions of the general economy
c) Expectations of a particular industry
d) Existing conditions of a particular industry
e) Competitive environment of a particular industry

2. Internal Factors

a) General expectations of the particular business being valued
b) Financial position/condition of the business being valued
c) Competitive position of the business being valued
d) Size of the business being valued
e) Nature of the business being valued
f) Quality and depth of the organization and staffing of the business being valued
g) Reliability or stability of the earnings of the business being valued

3. Investment Factors

a) Risk factors associated with the investment itself
b) Amount invested in the particular business–relative to other investments in the portfolio
c) Expectations of capital appreciation of the investment
d) Expectations of liquidity of the investment
e) Level of the expected management burden of the investment

Most valuation professionals agree that each of the above factors theoretically impacts the determination of an appropriate capitalization or discount rate. However, there remains no simple, generally accepted, or practical way to quantify these factors. Therefore, the determination of an appropriate capitalization or discount rate has been—and will continue to be—one of the most difficult and perplexing issues in the valuation process.

II. PROPER CAPITALIZATION/DISCOUNT RATES

A. CRITERIA

Two primary criteria exist for the determination of capitalization or discount rates in the context of valuing closely held businesses.

1. The capitalization or discount rate should be essentially the same as the rate of return (yield) that is currently being offered to attract capital or investment to the type, size, and financial condition of business that is being valued.
2. The capitalization or discount rate must be consistent with the "type" of benefit streams to be capitalized or discounted (e.g., pre-tax versus after-tax, cash flow vs. earnings to invested capital or equity).

B. CAPITALIZATION RATE = DISCOUNT RATE LESS LONG-TERM SUSTAINABLE GROWTH RATE

Observation
The term "earnings" as used in this book is synonymous with the term "benefit stream." These terms refer to cash flow, net income, or other types of benefit streams.

Once the analyst selects the appropriate type of earnings and estimates the amount of future earnings, an appropriate capitalization or discount rate must be determined. This rate is applied to the amount of estimated future earnings calculated. A capitalization rate is applied in a capitalization process to calculate value and a discount rate is applied in a discounting process to calculate value. For clarity, the rates are defined as follows:

1. **Discount rate:** A rate of return used to convert a series of future income amounts into their present value.
2. **Capitalization rate:** A divisor (or multiplier) used to convert a defined stream of income to a present indicated value.

It is generally accepted in the valuation community that subtracting a company's expected long-term sustainable growth rate from its discount rate yields the capitalization rate.

C. CAPITALIZATION VS. DISCOUNTING

A distinction between the capitalization process and the discounting process is the utilization of a terminal value. Recall that the discounting process calculates the present value of a series of forecasted future benefits. Forecasts are made for a finite number of future periods. Thus, when valuing a company using a discounting process, the analyst must consider terminal values. The terminal value represents the value of a company in the terminal year of an earnings forecast, or what the company will be worth in x number of years. There are several methods of estimating terminal value, including price/earnings and other multiples. The most frequently used method is to capitalize terminal year earnings using an appropriate capitalization rate and then discount the results back to a present value.

Recall that the capitalization rate is equal to the discount rate minus the projected growth rate. Thus, the discount rate and the capitalization rate are interchangeable only when there is no projected growth in the benefit stream. The following exhibits show the relationship between discounting and capitalizing future benefits under three future benefit assumptions:

1. The future benefit stream is linear and there is no growth.
2. The future benefit stream is linear but growing at a constant rate.
3. The future benefit stream reflects nonlinear growth.

Linear Benefits
Capitalization Rates vs. Discount Rates
No Growth

Assumptions	
Annual Benefits	$100,000
Discount Rate	20%
Growth	0%
Capitalization Rate	20%
End of period discounting convention	

DISCOUNTING

Year	1	2	3	4	5
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.8333	0.6944	0.5787	0.4823	0.4019
Discounted Benefits	83,333	69,444	57,870	48,225	40,188

Year	6	7	8	9	10
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.3348	0.2791	0.2326	0.1938	0.1615
Discounted Benefits	33,490	27,908	23,257	19,381	16,151

Year	11	12	13	14	15
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.1346	0.1122	0.0935	0.0779	0.0649
Discounted Benefits	13,459	11,216	9,346	7,789	6,491

Year	16	17	18	19	20
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.0541	0.0451	0.0376	0.0313	0.0261
Discounted Benefits	5,409	4,507	3,756	3,130	2,608

Year	21	22	23	24	25
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.0217	0.0181	0.0151	0.0126	0.0105
Discounted Benefits	2,174	1,811	1,509	1,258	1,048

Year	26	27	28	29	30
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.0087	0.0073	0.0061	0.0051	0.0042
Discounted Benefits	874	728	607	506	421

Year	31	32	33	34	35
Annual Benefits	$100,000	$100,000	$100,000	$100,000	$100,000
Discount Factor	0.0035	0.0029	0.0024	0.0020	0.0017
Discounted Benefits	351	293	244	203	169

Sum of the Discounted Benefits (rounded)	$499,200

CAPITALIZING METHODOLOGY

Annual Benefits	$100,000
Capitalization Rate	20.0%
Capitalized Benefits	$500,000

Conclusion: This calculation demonstrates that, with no growth, the capitalization process produces the same result as the discounting process.

Linear Benefits
Capitalization Rates vs. Discount Rates
With Growth

Assumptions	
Annual Benefits	$100,000
Discount Rate	25%
Growth	5%
Capitalization Rate	20%
End of period discounting convention	

DISCOUNTING

Year	1	2	3	4	5
Annual Benefits	$100,000	$105,000	$110,250	$115,763	$121,551
Discount Factor	0.8000	0.6400	0.5120	0.4096	0.3277
Discounted Benefits	80,000	67,200	56,448	47,417	39,830

Year	6	7	8	9	10
Annual Benefits	$127,629	$134,010	$140,711	$147,747	$155,134
Discount Factor	0.2621	0.2097	0.1677	0.1342	0.1074
Discounted Benefits	33,457	28,104	23,607	19,830	16,657

Year	11	12	13	14	15
Annual Benefits	$162,891	$171,036	$179,588	$188,567	$197,995
Discount Factor	0.0859	0.0687	0.0550	0.0440	0.0352
Discounted Benefits	13,992	11,754	9,873	8,293	6,966

Year	16	17	18	19	20
Annual Benefits	$207,895	$218,290	$229,205	$240,665	$252,698
Discount Factor	0.0281	0.0225	0.0180	0.0144	0.0115
Discounted Benefits	5,852	4,915	4,129	3,468	2,913

Year	21	22	23	24	25
Annual Benefits	$265,333	$278,600	$292,530	$307,157	$322,515
Discount Factor	0.0092	0.0074	0.0059	0.0047	0.0038
Discounted Benefits	2,447	2,056	1,727	1,451	1,218

Year	26	27	28	29	30
Annual Benefits	$338,641	$355,573	$373,352	$392,020	$411,621
Discount Factor	0.0030	0.0024	0.0019	0.0015	0.0012
Discounted Benefits	1,023	860	722	607	510

Year	31	32	33	34	35
Annual Benefits	$432,202	$453,812	$476,503	$500,328	$525,344
Discount Factor	0.0010	0.0008	0.0006	0.0005	0.0004
Discounted Benefits	428	360	302	254	213

Sum of the Benefits (rounded)	$498,900

CAPITALIZING

Annual Benefits	$100,000
Capitalization Rate	20%
Capitalized Benefits	$500,000

Conclusion: This calculation demonstrates that, with linear growth, the capitalization process produces the same result as the discounting process.

Nonlinear Growth
Capitalization Rates vs. Discount Rates

Assumptions					
	Year 1	Year 2	Year 3	Year 4	Year 5
Base Benefits	$100,000	$115,000	$143,750	$155,250	$186,300
Growth	15.0%	25.0%	8.0%	20.0%	3.0%
Annual Benefits	$115,000	$143,750	$155,250	$186.300	$191,889
Discount Rate	25.0%	25.0%	25.0%	25.0%	25.0%
Growth	15.0%	25.0%	8.0%	20.0%	3.0%
Capitalization Rate	10.0%	0.0%	17.0%	5.0%	22.0%
End of period discounting convention					

DISCOUNTING

Year	1	2	3	4	5
Annual Benefits	$ 115,000	$ 143,750	$ 155,250	$ 186,300	$ 191,889
Discount Factor	0.80000	0.64000	0.51200	0.40960	0.32768
Discounted Benefits	$ 92,000	$ 92,000	$ 79,488	$ 76,308	$ 62,878

Year	6	7	8	9	10
Annual Benefits	$ 197,646	$ 203,575	$ 209,682	$ 215,972	$ 222,451
Discount Factor	0.26214	0.20972	0.16777	0.13422	0.10737
Discounted Benefits	$ 51,812	$ 42,693	$ 35,179	$ 28,987	$ 23,885

Year	11	12	13	14	15
Annual Benefits	$ 229,125	$ 235,999	$ 243,079	$ 250,371	$ 257,882
Discount Factor	0.08590	0.06872	0.05498	0.04398	0.03518
Discounted Benefits	$ 19,682	$ 16,218	$ 13,363	$ 11,011	$ 9,073

Year	16	17	18	19	20
Annual Benefits	$ 265,618	$ 273,587	$ 281,795	$ 290,249	$ 298,956
Discount Factor	0.02815	0.02252	0.01801	0.01441	0.01153
Discounted Benefits	$ 7,476	$ 6,161	$ 5,076	$ 4,183	$ 3,447

Year	21	22	23	24	25
Annual Benefits	$ 307,925	$ 317,163	$ 326,678	$ 336,478	$ 346,572
Discount Factor	0.00922	0.00738	0.00590	0.00472	0.00378
Discounted Benefits	$ 2,840	$ 2,340	$ 1,928	$ 1,589	$ 1,309

Year	26	27	28	29	30
Annual Benefits	$ 356,969	$ 367,678	$ 378,708	$ 390,069	$ 401,771
Discount Factor	0.00302	0.00242	0.00193	0.00155	0.00124
Discounted Benefits	$ 1,079	$ 889	$ 733	$ 604	$ 497

Year	31	32	33	34	35
Annual Benefits	$ 413,824	$ 426,239	$ 439,026	$ 452,197	$ 465,763
Discount Factor	0.00099	0.00079	0.00063	0.00051	0.00041
Discounted Benefits	$ 410	$ 338	$ 278	$ 229	$ 189

Sum of the Discounted Benefits (rounded) $ 696,172

TWO STAGE MODEL

Year	1	2	3	4	5	Terminal
Annual Benefits	$ 115,000	$ 143,750	$ 155,250	$ 186,300	$ 191,889	$ 197,646
Capitalization Rate						22.0%
Capitalized Benefits						$ 898,391
Discount Factor	0.80000	0.64000	0.51200	0.40960	0.32768	0.32768
Discounted Benefits	$ 92,000	$ 92,000	$ 79,488	$ 76,308	$ 62,878	$ 294,385

Sum of discounted benefits and terminal value $ 697,059

Conclusion: This calculation demonstrates that, with nonlinear growth, the capitalizing terminal value process produces the same result as the discounting process.

III. IBBOTSON BUILD-UP METHOD

Observation

The capitalization/discount rate as used in business valuation is the expected yield rate on the investment.

It is extremely important that the analyst maintain consistency between the type of earnings and the capitalization or discount rates used in the valuation process. For example, a pre-tax rate should not be applied to net income because net income is assumed to be stated on an after-tax basis. This is a very simple distinction. However, often this distinction is overlooked in the valuation of a closely held business, thereby significantly over-valuing or under-valuing the business.

IBBOTSON BUILD-UP MODEL

Ibbotson Associates [Stocks, Bonds, Bills and Inflation (SBBI), Valuation Edition] provides a model that uses both historical data and current inputs to estimate the cost of equity capital for a company. The cost of capital is sometimes referred to as the expected or required rate of return.

Ibbotson's Build-Up[1] formula starts with the risk free rate and adds expected risk premiums designed to reflect the additional risk of an equity investment. The key variables used in estimating the cost of capital can be found in the 2005 SBBI Valuation Edition, Table 3-3, as follows:

The primary formula is: $K_e = R_f + ERP + IRP_i + SP + SCR$ *where:*

K_e	=	*cost of equity*
R_f	=	*risk free rate of return*
ERP	=	*expected equity risk premium, or the amount by which investors expect the future return on equity securities to exceed the risk free rate*
IRP_i	=	*expected industry risk premium for industry i reflecting the relative risk of companies in that industry (if appropriate)*
SP	=	*size premium*
SCR	=	*specific company risk for the company*

[1] One of multiple methods available in BVMPro capitalization rates section as one method a valuation analyst may select.

The table[2] shown below is reproduced from Ibbotson's SBBI: Valuation Edition 2005 Yearbook, Table C-1 and will change according to data gathered in any given year.

KEY VARIABLES IN ESTIMATING THE COST OF CAPITAL

	Value
Yields (Riskless Rates) [3]	
Long-term (20-year) U.S. Treasury Coupon Bond Yield	4.8%
Equity Risk Premium [4]	
Long-horizon expected equity risk premium (historical): large company stock total returns minus long-term government bond income returns	7.2
Long-horizon expected equity risk premium (supply side): historical equity risk premium minus price-to-earnings ratio calculated using three-year average earnings	6.1

Size Premium[5]

Decile	Market Capitalization of Smallest Company (in millions)		Market Capitalization of Largest Company (in millions)	Size Premium (Return in Excess of CAPM)
Mid-Cap, 3-5	$1,607.931	-	$6,241.953	0.95%
Low-Cap, 6-8	$506.410	-	$1,607.854	1.81
Micro Cap, 9-10	$1.393	-	$505.437	4.02
Breakdown of Deciles 1-10				
1- Largest	$14,099.878	-	$342,087.219	-0.37
2	$6,258.530	-	$14,096.886	0.60
3	$3,473.335	-	$6,241.953	0.75
4	$2,234.146	-	$3,464.104	1.07
5	$1,607.931	-	$2,231.707	1.44
6	$1,098.284	-	$1,607.854	1.75
7	$746.249	-	$1,097.603	1.61
8	$506.410	-	$746.219	2.36
9	$262.974	-	$505.437	2.86
10-Smallest	$1.393	-	$262.725	6.41
Breakdown of the 10th Decile				
10a	$144.122	-	$262.725	4.54
10b	$1.393	-	$143.916	9.90

[2] Used with permission, *SBBI: Valuation Edition 2005 Yearbook*, updated annually; all rights reserved.
[3] As of December 31, 2004. Maturity is approximate.
[4] See Chapter 5 of *SBBI Valuation Edition 2005 Yearbook* for complete methodology.
[5] See Chapter 7 of *SBBI Valuation Edition 2005 Yearbook* for complete methodology.

A. EQUITY RISK PREMIUM

The historical equity risk premium shown in the table above is calculated as the return an investor would have received on the S&P 500 in excess of the return on Treasury securities, during the period from 1926 through the present.

Why focus on the long-term period? Ibbotson offers the following observations:

1. Long-term historical returns have shown surprising stability.
2. Short-term observations may lead to illogical forecasts.
3. Focusing on the recent past ignores dramatic historical events and their impact on market returns. We don't know what major events lie ahead.
4. Law of large numbers: more observations lead to a more accurate estimate.

In addition to these observations, another justification for using long-term data is that investments in closely held businesses generally represent long-term investments. Thus, uses of Ibbotson's equity risk premium are more likely to match investment horizons than premiums calculated with short-term data.

Inherent in this discussion is the assumption that past returns provide a valid estimate of current (and future) cost of capital. Recent research suggests this assumption may be invalid. Ibbotson notes there have been a recent (over the past 20 years) increase in the average price to earnings ratio (P/E), and this increase accounts for part of the historical equity risk premium. Since similar increases in P/E ratio are not expected, future equity risk premiums are expected to be lower. This lower expected premium can be seen in the "supply side" equity risk premium calculation in the table above.[6]

Although recent research raises questions about whether the equity risk premium should be reduced based upon this "supply side" argument, Ibbotson does not recommend making this adjustment. The following is quoted from Ibbotson's *2006 SBBI Valuation Edition*, pages 92 to 98:

> *"Long-term expected equity returns can be forecasted by the use of supply side models. The supply of stock market returns is generated by the productivity of the corporations in the real economy. Investors should not expect a much higher or lower return than that produced by the companies in the real economy. Thus, over the long run, equity returns should be close to the long-run supply estimate.*
>
> *From the end of 1925 to the end of 2005, the overall stock market price grew faster than GDP per capita. This is primarily because the price-to-earnings ratio increased 1.74 times during the same period.*
>
> *As mentioned earlier, one of the key findings of the Ibbotson and Chen study is that* P/E *increases account for only a small portion of the total return of equity (0.65% of the total 10.36%). The reason we present supply side equity risk premium going back only 20 years is because the P/E ratio rose dramatically over this time period, which caused the growth rate in the P/E ratio calculated from 1926 to be relatively high. The subtraction of the P/E growth factor from equity returns has been responsible for the downward adjustment in the supply side equity risk premium compared to the*

[6] For a more thorough discussion of this and other possible adjustments to the historical equity risk premium, see Ibbotson's *SBBI Valuation Edition* and Ibbotson and Chen's *Stock Market Returns in the Long Run: Participating in the Real Economy*.

historical estimate. Beyond the last 20 years, the growth factor in the P/E *ratio has not been dramatic enough to require an adjustment.*

This section has briefly reviewed some of the more common arguments that seek to reduce the equity risk premium. While some of these theories are compelling in an academic framework, most do little to prove that the equity risk premium is too high. When examining these theories, it is important to remember that the equity risk premium data outlined in this book (both the historical and supply side estimates) are from actual market statistics over a long historical time period."

B. SIZE PREMIUM

The correlation between company size and return has been well documented by Ibbotson and other researchers. Over long periods of time, returns on investments in smaller firms have consistently and significantly exceeded returns on investment in larger firms. The size premium is the extra return a willing investor would expect to receive by investing in smaller equity securities on the NYSE/AMEX/NASDAQ over the large equity security. Since virtually all closely held companies are smaller than even the smallest of the S&P 500 companies examined by Ibbotson, an analyst should consider the inclusion of a size premium in the build-up model.

Long-term returns for all publicly traded stocks are calculated in Ibbotson's *SBBI Valuation Edition*. These returns are then ranked into deciles based on company size. The resulting table (shown on the previous page) clearly illustrates that average returns for small publicly traded companies have been consistently and significantly higher than average returns for large corporations. Since the typical closely held business would fall into the tenth decile in terms of size, the risk premium for this decile is of great interest to the valuation analyst.

To gain greater insight into the small stock risk premium, Ibbotson splits the tenth decile (containing the smallest companies) in half, calculating returns on the smallest five percent (decile 10b) and second smallest five percent (decile 10a) of public companies. The results are striking. As can be seen in the exhibit, the size premium for the smallest five percent (10b) is 9.90 percent, more than double the premium for the 10a companies. This suggests that the risk premiums for very small companies may be significantly higher than previously recognized.[7]

C. INDUSTRY RISK PREMIUM

Ibbotson's general equity risk premium and size premia are not industry specific. Since some industries are inherently riskier than others, inclusion of an industry specific risk premium can result in a more precise estimate of the cost of capital.

Ibbotson has developed an industry premium methodology that valuators may now reference and cite in their valuation reports. This methodology relies on the full information beta estimation process outlined in the *SBBI Valuation Edition Yearbook*. The full information beta methodology uses data from companies participating in an industry to evaluate the risk characteristics of that industry. The full information approach provides a risk index for each industry. The risk index compares the risk level of a specific industry to the total market.

[7] A comprehensive discussion of the statistical relationship between size and historical returns can be found in *Ibbotson's SBBI Valuation Edition*, Chapter 7.

Only industries with full information beta were included in the analysis, with a minimum of five companies in each industry. The equation is as follows:

$IRP_i = (RI_i \times ERP) - ERP$ where:

IRP_i	=	The expected industry risk premium for industry i, or the amount by which investors expect the future return of the industry to exceed that of the market as a whole
RI_i	=	The risk index for industry i
ERP	=	The expected equity risk premium

Source: *SBBI Valuation Yearbook*, Chapter Three, The Buildup Method, Industry Premia.

Table 3-5 in the *SBBI Valuation Yearbook*, Chapter Three, Industry Premia Estimates provides the valuator with industry premia by SIC code.

In addition, this additional risk premium or discount may be determined by focusing on how the general economy compares with expectations for the particular industry. Key questions include: How has this industry reacted to similar general economic conditions in the past? What are the industry forecasts and how do they relate to this company? What is its position in the industry? In addition to answering the aforementioned questions, it is necessary to compare the financial analysis of the company to the industry financial analysis; and finally, to assess additional company specific risk based on the financial analysis of the company.

D. SPECIFIC COMPANY RISK PREMIUM

The final variable in Ibbotson's Build-up model addresses company-specific risk factors. If used correctly, the previous four factors (risk free rate, equity risk premium, size premium and industry premium) should yield the estimated cost of capital for an equity investment in a smaller, typical company in the identified industry. To assume that this estimated cost of capital is appropriate for the analyst's company would be to ignore possibly critical aspects of that company.

For example, the target company could be relatively new, or it could have a lengthy record of strong performance and a dominant position in its market. Other characteristics, such as poor planning, the quality of management, lack of capital, access to debt and inadequate business experience must be considered. A thorough analysis of the company's risk ratios and how they compare with industry norms can help identify these company-specific risks.

The specific company risk described above is referred to as "unsystematic risk". This risk measures the uncertainty of returns arising from characteristics of the industry and the individual company. In a well-balanced economic portfolio, the unsystematic risk can be eliminated through diversification. This is not the case with an investment in one closely held company's stock.

In evaluating company-specific risks, the authors of Practitioners Publishing Company's *Guide to Business Valuation* suggest that the following factors be considered in the specific company risk premium:

1. The Company's Financial Risk

The term financial risk is defined broadly in this context to include not only risks from debt financing, but also the relative risk from all means of financing the business. This would include current liabilities and the choice to liquidate non-cash assets into cash to finance capital investment or pay a dividend. An assessment of financial risk therefore involves all of the following:[8]

a) Interest-bearing leverage and coverage ratios
b) Total leverage ratios, such as total liabilities to equity
c) Liquidity ratios, such as the current and quick ratio
d) Volatility of earnings:
   Forecasting future earnings growth may add an additional risk premium to the calculations of discount rate. Estimating growth in earnings should be undertaken only in situations where the analyst has strong reason to believe there is a high likelihood of continued growth (see Chapter Four). If this is the case, then much of the risk of forecasting growth is eliminated.
e) Turnover ratios, such as inventory and receivables turnover

A company that runs too lean, or is too highly leveraged with debt, will generally be riskier than a company that is not so highly burdened.

2. The Diversification of the Company's Operations

Generally, the more diversified a company is in terms of products, customer base, geographic locations, etc., the less the risk compared to other companies.

3. Other Operational Characteristics

The analyst should also assess all other factors that could lead to additional positive or negative adjustments. Such factors often include key-man issues and management depth and competence.

E. GROWTH RATE SHOULD EQUAL INFLATION PLUS REAL GROWTH THAT CAN BE ACHIEVED WITHOUT ADDITIONAL CAPITAL INVESTMENT

It is generally accepted that an Expected Long-Term Average Growth Rate is impossible to sustain into perpetuity if it exceeds inflation plus population growth. The rate does not include growth in overall company cash flows dependent on future capital investment. A common error is to use a rate of growth that could not be achieved without additional capital investment(s). Often, this is related to the position of the company in its life cycle. What is its state of maturity? Is it experiencing rapid growth, slow growth, stagnation or decline?

Capitalization models are inherently sensitive to the choice of growth rate, and the analyst should be careful to select a rate that is reasonable. Remember, this is not a short-term growth rate, this must be a long-term sustainable growth rate! To demonstrate how sensitive the model is, consider a company with normalized earnings of $100,000. Assuming Ibbotson's build-up model yields a cost of equity capital of 20 percent, the use of a three percent growth rate will

[8] See Practitioners Publishing Company's *Guide to Business Valuation, 15th Edition.*

result in a conclusion of value of $588,235. However, use of a more aggressive six percent perpetual growth rate results in a conclusion of value of $714,286, more than 21 percent higher.

The analyst should be careful to select a rate that is reasonable, particularly when using business valuation software which may default to the company's historical growth rate. Many valuators believe the long-term sustainable growth rate for mature companies should be in the range of three to four percent.

ILLUSTRATION
IBBOTSON BUILD-UP METHOD

Risk-free long-term U.S. Government bond rate			+	5.22%	Note A
Equity risk premium	+	7.20%			Note B
Size premium	+	6.41%			Note C
Industry premium (can be positive or negative)	+	0.63%			Note E
Return in excess of risk-free rate	=		+	14.24%	
Total risk premium for company specific risk	=		+	5.50%	Note D
After-tax net cash flow discount rate (This discount rate would be used to calculate the present value of cash flows to equity)			=	24.96%	
Long-term sustainable growth rate			-	3.00%	Note F
After-tax net cash flow capitalization rate for next year			=	21.96%	
Adjustment to current year (one plus growth rate)			÷	1.03	
After-tax net cash flow capitalization rate for current year (This capitalization rate would be suitable for determining the value of the future cash flow stream in the capitalization model)			=	21.32%	
Cash to earnings factor			+	3.37%	Note G
After-tax net income capitalization rate for the current year			=	24.69%	
Intangible earnings factor			+	5.00%	Note H
After-tax intangible capitalization rate for the current year			=	29.69%	
Tax effect [1-tax rate (40%)]			÷	60.00%	
Pre-tax net income capitalization rate for the current year			=	41.15%	
Pre-tax intangible capitalization rate for the current year			=	49.48%	

Note A	20-year yield to maturity on U.S. government bonds at the valuation date, from *Wall Street Journal* or *St. Louis Federal Reserve* or other source.
Note B	Long-horizon expected equity risk premium (historical rate), *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note C	Size premium for Decile 10 from Appendix C-1, *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note D	Subjective risk premium for company-specific risks.
Note E	Industry risk premium estimate for SIC 1799, Specialty Trade Contractors, from Table 3-5 of *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note F	Long-term sustainable growth rate of economic equity returns based on industry outlook and discussions with management.
Note G	Increment to convert to net earnings; EPS less dividend per share, or company's actual increment.
Note H	Additional subjective risk premium associated with intangible earnings.

F. CALCULATION OF CASH TO EARNINGS FACTOR

When future earnings approximate future cash flows, no adjustment is necessary to convert the capitalization rate (or discount rate) for cash flows into a capitalization rate (or discount rate) for accrual earnings. However, when the analyst expects that future cash flows will **not** be consistent with future earnings, adjustment of the cash flow capitalization and discount rates is necessary. The following is one methodology used to determine the cash to earnings factor:

	Earnings	*Depr*	*Working Capital*	*CapX*	*Debt*	*Cash Flow*	*Factor*
Prior Yr.	*$ 3,948,781*	*$ 248,626*	*$ (395,000)*	*$ (529,336)*	*$ -*	*$ 3,273,071*	*82.89%*
2nd Prior Yr.	*2,010,629*	*309,669*	*(201,000)*	*(13,130)*	*-*	*2,106,168*	*104.75%*
3rd Prior Yr.	*5,499,938*	*317,066*	*(550,000)*	*(227,431)*	*-*	*5,039,573*	*91.63%*
4th Prior Yr.	*3,132,499*	*321,356*	*(313,000)*	*(138,137)*	*(45,103)*	*2,957,615*	*94.42%*
5th Prior Yr.	*1,641,937*	*310,768*	*(164,000)*	*(286,059)*	*45,103*	*1,547,749*	*94.26%*
6th Prior Yr.	*837,851*	*291,189*	*(84,000)*	*(317,588)*	*(406,629)*	*320,823*	*38.29%*
7th Prior Yr.	*1,844,016*	*233,631*	*(184,000)*	*(672,544)*	*(128,955)*	*1,092,148*	*59.23%*
	$18,915,651	*$2,032,305*	*$(1,891,000)*	*$(2,184,225)*	*$(535,584)*	*$16,337,147*	*86.37%*
Avgs	*$ 2,700,000*	*$ 290,000*	*$ (270,000)*	*$ (310,000)*	*$ (80,000)*	*$ 2,330,000*	

After-tax net income capitalization rate for the current year (21.32%/86.37%)	*24.69%*
After-tax net cash flow capitalization rate for the current year	*21.32%*
Cash to earnings factor	*3.37%*

Proof

	Earnings	*Cash Flow*
Benefit Stream	*$ 2,700,000*	*$ 2,330,000*
Capitalization Rate	*24.69%*	*21.32%*
Enterprise Value (rounded)	*$10,900,000*	*$10,900,000*

In *The Cost of Capital*, Pratt advises:

> *"Another way of looking at cash flow would be to define it more broadly. Instead of considering only the cash flows investors actually receive, you might define net cash flows as those amounts that could be paid to equity investors without impeding a company's future growth. Of course these cash flows are not those paid to investors, but presumably, investors will ultimately realize the benefit of these amounts either through higher future dividends or, more likely, stock appreciation. Some analysts assume that over the long run, net (after-tax) income should be quite close to cash flow. Therefore, they assume that net income can be used as a proxy for net cash flow. This assumption should be questioned on a case-by-case basis."*

IV. CAPITAL ASSET PRICING MODEL (CAPM)

CAPM, by definition, is an "equilibrium asset pricing theory that shows that equilibrium rates of expected return on all risky assets are a function of their co-variance with the market portfolio."

This method for determining a capitalization or discount rate is based on the theory that investors in risky assets require a rate of return above and beyond a risk free rate as compensation for bearing the risk associated with holding the investment.

A. ASSUMPTIONS

The assumptions underlying the capital asset pricing model are:

1. Investors are risk averse.
2. Rational investors seek to hold portfolios which are fully diversified.
3. All investors have identical investment holding periods.
4. All investors have the same expectations regarding expected rate of return, and how capitalization rates are generated.
5. There are no transaction costs.
6. There are no taxes.
7. The rate received from lending money is the same as the cost of borrowing.
8. The market has perfect diversity and liquidity so an investor can readily buy or sell any fractional interest.

B. CALCULATION OF EXPECTED RETURN

Expected return = Risk-free rate + Beta x { Expected return on a market portfolio — Risk-free Rate }

Abbreviated, the variables and the equation appear as follows:

$ER_i = R_f + B\ (ER_m - R_f)$

The risk-free (R_f) rate is represented by the 20-year yield to maturity on US Government bonds. According to Ibbotson, "the horizon of the chosen Treasury security should match the horizon of whatever is being valued. When valuing a business that is being treated as a going concern, the appropriate Treasury yield should be that of a long-term Treasury bond." The expected return on a market portfolio (R_m) is the actual return on the Standard and Poor's 500 (S&P 500) Index. The beta coefficient (*B*) is a key variable in the CAPM equation. In the standard CAPM calculation, it represents the co-variance of the rate of return on the subject security, with the rate of return on the market divided by the variance of the market. More simply, it is a measure of the volatility of the subject security as compared to the market.

In order to fully understand beta, certain terms must be understood:

1. Variance is a measure of the squared deviation of the actual return of a security from its expected return.
2. Co-variance is a statistical measure of the interrelationship between two securities.

In the standard calculation of CAPM, beta is computed using the return on investment (ROI) of the subject security. Since ROI is calculated using the price of stock, the analyst uses the standard CAPM very rarely. If the price of stock is known, is there a need for valuation?

Some analysts alter the CAPM model by modifying certain variables. The risk-free rate (R_f) is represented by the intermediate term (five to 10 year) Treasury bond yield rate. Beta (*B*) is modified so that it represents the co-variance of the pre-tax return on equity (ROE) of the subject company, with the ROE of other specific companies or industry averages divided by the variance of the ROE of the industry. Finally, rather than using the expected return on a market portfolio as the ER_m, it is represented by the average pre-tax ROE of the specific companies or the industry in which the subject company operates.

1. Calculation of Beta (β)

   A beta[9] of 1.0 would indicate the subject company is no more or no less volatile than the industry. In this example the beta of 0.8501 indicates that the subject company is less volatile than the industry. As such, it would appear to be a better risk. Thus, a total risk-premium less than the industry would probably be appropriate for the company. Based on this analysis, it can be seen that the expected rate of return for a company should be positively related to its beta.

2. Security Market Line (SML)

   The expected return on a security with a beta of zero is the risk-free rate, since a zero beta indicates no relative risk. The expected return on a security with a beta of one is the expected return of the market, since a beta of one indicates that the security has the same relative risk as the market.

   A shortcoming of CAPM is the fact that it utilizes comparative information in its various forms. Since it may be extremely difficult to locate industry data, it may be difficult to use CAPM to develop a discount/capitalization rate. It is equally as difficult to find specific comparable company data for a closely held company.

3. Is CAPM a pre-tax or an after-tax method? The answer: it depends.

   CAPM describes the cost of equity for a given company, and is equal to the risk-free rate plus some amount to compensate for the risk involved in excess of the risk-free rate. Thus, there are several elements to CAPM coming from both sides of the tax equation. This risk-free rate is usually a government bond rate, which is pre-tax to the investor. The expected return on a market portfolio is generated from average returns of the market after corporate tax, usually comparing the return to that of the S&P 500. Beta is public market volatility, generated by stock transactions, which is after corporate tax (but again, pre-investor tax). These companies' 10K forms do consider known tax liabilities in their bottom lines. However, this liability may not be the actual tax. In valuing a closely held company, beta is generally developed from comparable public companies or is calculated using the average pre-tax ROE (for equity capital) or ROI (for investment) of the specific company. ROI, as used to develop beta[10], is calculated as:

$$\frac{\text{(Ending Stock Price - Beginning Stock Price) + Dividends}}{\text{Beginning Stock Price}}$$

[9] Historical beta research can be performed by KeyValueData.

[10] β or *b* often (but not always) indicate beta in financial equations.

This generates an after-tax rate (or variable) as the capacity to pay dividends (a key element) is based on after-tax earnings.

When the analyst uses CAPM to generate a capitalization rate, the risk rate for the general public market is an after-tax rate; therefore, CAPM is an after-tax method. Ibbotson considers its build-up method, loosely based on CAPM, to be an after-tax calculation. If the analyst uses RMA's ROE, it is pre-tax. One must be certain to identify variables when using CAPM to quantify a capitalization/discount rate.

ILLUSTRATION
MODIFIED CAPM IBBOTSON BUILD-UP METHOD

Risk-free long-term U.S. Government bond rate			+	5.22%	Note A
Equity risk premium	+	7.20%			Note B
Beta	x	1.511			Note B1
Average company comparative return	=	10.88%			
Size premium	+	6.41%			Note C
Return in excess of risk-free rate	=		+	17.29%	
Total risk premium for company specific risk			+	2.45%	Note D
After-tax net cash flow discount rate (This discount rate would be used to calculate PV of cash flows to equity)			=	24.96%	
Long-term sustainable growth rate			-	3.00 %	Note F
After-tax net cash flow capitalization rate for next year			=	21.96%	
Adjustment to current year (one plus growth rate)			÷	1.03%	
After-tax net cash flow capitalization rate for the current year (This capitalization rate would be suitable for determining the value of future cash flow stream with constant growth)			=	21.32%	
Cash to earnings factor			+	3.37%	Note G
After-tax net income capitalization rate for the current year			=	24.69%	
Intangible earnings factor			+	5.00%	Note H
After-tax intangible capitalization rate for the current year			=	29.69%	
Tax effect [1-tax rate (40%)]			÷	60.00%	
Pre-tax net income capitalization rate for the current year			=	41.15%	
Pre-tax intangible capitalization rate for the current year			=	49.48%	

Note A	20-year yield to maturity on U.S. government bonds at the valuation date, from *Wall Street Journal* or *St. Louis Federal Reserve* or other source.
Note B	Long-horizon expected equity risk premium (historical rate), from *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note B1	Comparative company beta from selected guideline companies or comparative industry beta from *Cost of Capital Quarterly*, Ibbotson Associates, Inc.
Note C	Size premium for Decile 10 from Appendix C-1, *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note D	Subjective risk premium for company-specific risks.
Note E	Industry risk premium estimate for SIC 1799, Specialty Trade Contractors, from Table 3-5 of *Stocks, Bonds, Bills, and Inflation: Valuation Edition*, Ibbotson Associates, Inc.
Note F	Long-term sustainable growth rate of economic equity returns based on industry outlook and discussions with management.
Note G	Increment to convert to net earnings; EPS less dividend per share, or company's actual increment.
Note H	Additional subjective risk premium associated with intangible earnings.

Observation
No matter the individual components of the required rate of return, it is important to keep in mind that the resulting rate is intended to attract an investor to the investment. As such, you may capture certain elements of risk in various components (i.e. ERP, Beta, etc.). In the end, the rate as a whole must make sense given the risks attributable to the investment, and the facts and circumstances unique to each case.

ILLUSTRATION
IBBOTSON BUILD-UP METHOD COMPARED TO MODIFIED CAPM

		Build-Up		***Modified CAPM***		***Modified Build-Up***	*A*
Risk-free long-term US Government bond rate	+	*5.22%*	+	*5.22%*	+	*5.22%*	
Equity risk premium		*7.20%*		*7.20%*		*7.20%*	
Beta			*x*	*1.511*			
Average company comparative return	+	*12.42%*	+	*16.10%*	+	*12.42%*	
Size premium	+	*6.41%*	+	*6.41%*	+	*9.34%*	*A*
Industry premium (can be positive or negative)	+	*0.63%*		*0.00%*	+	*0.63%*	
Return in excess of risk-free rate	=	*19.46%*	=	*22.51%*	=	*22.39%*	
Total risk premium for company specific risk	+	*5.50%*	+	*2.45%*	+	*2.57%*	
After-tax net cash flow discount rate (This discount rate would be used to calculate PV of cash flows to equity)	=	*24.96%*	=	*24.96%*	=	*24.96%*	
Long-term sustainable growth rate	-	*3.00%*	-	*3.00%*	-	*3.00%*	
After-tax net cash flow capitalization rate for next year (This capitalization rate would be suitable for determining the terminal value)	=	*21.96%*	=	*21.96%*	=	*21.96%*	
Adjustment to current year (one plus growth rate)	÷	*1.03*	÷	*1.03*	÷	*1.03%*	
After-tax net cash flow capitalization rate for current year (This capitalization rate would be suitable for determining the value of future cash flow stream with constant growth)	=	*21.32%*	=	*21.32%*	=	*21.32%*	
Cash to earnings factor	+	*3.37%*	+	*3.37%*	+	*3.37%*	
After-tax net income capitalization rate for the current year	=	*24.69%*	=	*24.69%*	=	*24.69%*	
Intangible earnings factor	+	*5.00%*	+	*5.00%*	+	*5.00%*	
After-tax intangible capitalization rate for the current year	=	*29.69%*	=	*29.69%*	=	*29.69%*	
Tax effect [(1-tax rate (40%)]	÷	*60.00%*	÷	*60.00%*	÷	*60.00%*	
Pre-tax net income capitalization rate for the current year	=	*41.15%*	=	*41.15%*	=	*41.15%*	
Pre-tax intangible capitalization rate for the current year	=	*49.48%*	=	*49.48%*	=	*49.48%*	

Note A *Modified build-up represents the size premium utilized in the equity risk premium unadjusted by beta. Whereas, the straight build-up represents the size premium in excess of CAPM.*

V. EQUITY RISK PREMIUM: DUFF & PHELPS, LLC RISK PREMIUM REPORT

Another source of equity risk premiums (ERPs) used in the development of discount and capitalization rates can be found in the annual *Risk Premium Report* (the "Report") published by Duff & Phelps, LLC. This report was formerly known as the *Standard & Poor's Corporate Value Consulting Risk Premium Report* and is based on research conducted by Roger Grabowski and David King. The Report is published annually and can be purchased from Morningstar at their website http://corporate.morningstar.com/ib/asp/subject.aspx?xmlfile=1425.xml. An example of the 2005 report is located in Appendix X.

The Duff & Phelps ERP measurements are based on company information from the Center for Research in Security Prices (CRSP) database and the Standard & Poor's Compustat database. The study begins with 1963, the year the Standard & Poor's Compustat database was established. The Report consists of two parts; Part I presents data related to historical equity risk premiums and company size and Part II presents data quantifying the relationship between historical equity risk premiums and company risk.

A. COMPANIES INCLUDED IN THE DATA

Companies included in the measurement data must meet certain criteria including the following:

1. Must be included in both the CRSP and the Compustat databases
2. Excludes financial service companies (Standard Industrial Classification = 6)
3. Must be publicly traded for 5 years
4. Must have sales greater than $1 million in any of the previous 5 years
5. Must have a positive 5-year average earnings before interest, taxes, depreciation and amortization (EBITDA) for the previous five fiscal years

Duff & Phelps also created a separate "high financial risk" portfolio consisting of companies:

1. Identified by Compustat as in bankruptcy or liquidation
2. With 5-year average net income available to common equity for the previous five years less than zero
3. With 5-year average operating income for the previous five years less than zero
4. With negative book value of equity at any of the previous five fiscal year-ends
5. With debt-to-total capital of more than 80%

B. SIZE MEASUREMENT

Company data is sorted by eight measures of size and each measurement of size is included as a separate exhibit in the Report. The measures of size include:

1. Market value of common equity (common stock price times number of common shares outstanding)
2. Book value of common equity (does not add back the deferred tax balance)
3. 5-year average net income for previous five fiscal years (net income before extraordinary items)
4. Market value of invested capital (market value of common equity plus carrying value of preferred stock plus long-term debt (including current portion) and notes payable)
5. Total assets (as reported on the balance sheet)
6. 5-year average EBITDA for the previous five fiscal years
7. Sales (net)
8. Number of employees (either at year-end or yearly average, including part-time and seasonal workers)

Companies that meet the criteria noted above are then divided evenly into twenty-five portfolios for each measure of size. Companies included in the high financial risk portfolio are shown as a separate line item in each of the size categories.

C. DATA PRESENTATION

The Duff & Phelps data are presented in a series of exhibits in Appendix X.

Part I of the Report includes:

Exhibits A-1 through A-8	ERP vs. company size (eight measures of size)
Exhibits B-1 through B-8	Premiums over Capital Asset Pricing Model (CAPM) vs. company size (eight measures of size)

Part II of the Report includes:

Exhibits C-1 through C-8	Relation between size and company risk (eight measures of size)
Exhibits D-1 through D-3	ERP vs. company risk (three measures of risk)

The three company risk measures are as follows:

- Operating margin (the lower the margin, the greater the risk)
- Coefficient of Variation in Operating Margin (the greater the coefficient of variation, the greater the risk)
- Coefficient of Variation in Return on Equity (the greater the coefficient of variation, the greater the risk)

D. DATA USE

The ERPs developed by the Duff & Phelps data can be used to calculate a discount cost of equity using a build-up model (using the data reported in Exhibits A-1 through A-8) or the modified capital asset pricing model (MCAPM) (using the data reported in Exhibits B-1 through B-8).

The Report suggests that the "smoothed" average premium is the most appropriate indicator for most of the portfolio groups. The "smoothed" premium refers to how the premium is determined. It can be calculated based on a regression analysis, with the average historical ERP as the dependent variable and the logarithm of the average sorting criteria as the independent variable. One benefit of the "smoothed" premium is if an analyst is estimating the required rate of return for a company significantly smaller than any of the companies found in the smallest of the 25 portfolios, it is appropriate to extrapolate the ERP using the slope and constant terms from the regression relationships used in deriving the "smoothed" premiums.

E. BUILD-UP METHOD EXAMPLE

Using the build-up method to determine a required rate of return on equity, assume the subject company has the following characteristics:

Eight Measures of Size	***Amount***
Market value of equity	*$120 million*
Book value of equity	*$100 million*
5-year average net income	*$10 million*
Market value of invested capital	*$180 million*
Total assets	*$300 million*
5-year average EBITDA	*$30 million*
Sales	*$250 million*
Number of employees	*200*

Using each of the exhibits A-1 through A-8 (for each of the size measurements) we extract the following ERP data:

Eight Measures of Size	***Company Size***	***Exhibit***	***Guideline Portfolio***	***Smoothed Average ERP ****
Market value of equity	*$120 million*	*A-1*	*24*	*12.3%*
Book value of equity	*$100 million*	*A-2*	*24*	*11.3%*
5-year average net income	*$10 million*	*A-3*	*23*	*11.4%*
Market value of invested capital	*$180 million*	*A-4*	*24*	*12.0%*
Total assets	*$300 million*	*A-5*	*23*	*11.2%*
5-year average EBITDA	*$30 million*	*A-6*	*24*	*11.8%*
Sales	*$250 million*	*A-7*	*23*	*11.1%*
Number of employees	*200*	*A-8*	*25*	*12.6%*
Mean				*11.7%*
Median				*11.6%*

* over the riskless rate

The Report states that it has used the Ibbotson Associates' income return on long-term Treasury bonds as their measure of the historical riskless rate, therefore a 20-year Treasury bond yield is the most appropriate measure of the riskless rate to use with the Duff & Phelps ERPs.

Thus, if we have a riskless rate of 4.7% as of the valuation date, the Duff & Phelps data would indicate a required rate of return on equity ranging from 15.8% to 17.3%, with an average of 16.4%. From this point, the valuator needs to consider the company specific risk factor. For more, refer to discussion on this subject earlier in this Chapter.

Observation

As with all other methodologies presented in this course, it is important to acquire and read the underlying analysis and supporting data provided in the Duff & Phelps report before using the data.

VI. WEIGHTED AVERAGE COST OF CAPITAL (WACC)

Another calculation used to develop a discount or capitalization rate is known as the weighted average cost of capital or WACC. A company's capital structure may consist of the following, in any combination:

1. Common Equity
2. Preferred Equity
3. Long-term Debt

As its name implies, WACC actually blends a company's cost of equity with its cost of debt to arrive at the company's overall cost of capital. WACC is used when the valuation analyst wants to determine the value of the entire capital structure of a company, such as in an acquisition scenario.

WACC adds versatility to the valuation in that it can be developed based on a number of assumptions involving the company's debt in its capital structure. These assumptions can include greater debt, less debt, or debt under different terms.

A. CALCULATION OF THE WEIGHTED AVERAGE COST OF CAPITAL

Assuming a simple capital structure consisting only of common equity and long-term debt, the formula to develop WACC is as follows:

$WACC = (k_e \times W_e) + (k_{d/(pt)} [1\text{-}t] \times W_d)$ where:

WACC	=	Weighted Average Cost of Capital
k_e	=	Cost of common equity capital
W_e	=	Percentage of common equity in the capital structure, *at market value*
$K_{d/(pt)}$	=	Cost of debt capital (pre-tax) for the company
t	=	Effective income tax rate for the company
W_d	=	Percentage of debt in the capital structure, *at market value*

Note that if the capital structure includes preferred equity, the formula would change to reflect the third component as follows:

$$WACC = (k_p \times W_p) + (k_e \times W_e) + (k_{d/(pt)} [1\text{-}t] \times W_d)$$

Where k_p is the cost of preferred equity and W_p is the percentage of preferred equity in the capital structure at market time.

The WACC as computed is an "after-tax WACC," as it is normally applied to cash flows after entity-level taxes.

An important point to note in calculating the WACC for a privately-held company is that since no market value exists for the capital structure weightings, the analyst must *estimate* the market values in order to eventually arrive at their market value. Another point to note is that the analyst will typically assume that the book value of the debt approximates its market value, particularly if the debt is from a third-party institution (i.e., bank).

Calculation of the WACC for a privately-held company is a circular process and can be illustrated as follows:

Example:

Assume the following information applies to Terra Company:

- *Book value* of long-term debt = $300,000 (30%)
- *Book value* of common equity = $700,000 (70%)
- Interest rate on the long-term debt = 5.0%
- Cost of equity (using a build-up method) = 22%
- Effective tax rate = 40%
- Net cash flow to invested capital = $250,000

First Iteration

The analyst must first *estimate* the market values of the capital structure weightings and include the estimations in the formula. For this example, the book values are the first estimate of the market value weights. Applying the estimates to the WACC formula, the result is as follows:

$$
\begin{aligned}
\text{WACC} &= (k_e \times W_e) + (k_{d/(pt)}\,[1\text{-}t] \times W_d) \\
&= (0.22 \times 0.70) + (0.05\,[1 - 0.40] \times 0.30) \\
&= (0.154) + (0.03 \times 0.30) \\
&= 0.154 + 0.009 \\
&= 0.163 \\
&= 16.3\%
\end{aligned}
$$

Proof – First Iteration

With the first iteration resulting in a WACC of 16.3%, the analyst then applies this to the net cash flow to invested capital to calculate a value. For this example a capitalization valuation model is used although a discounting valuation model could also be used. Using an assumed growth rate of 3.0%, the proof calculation is as follows:

$$
\begin{aligned}
\text{Estimated value} &= \text{Net cash flow to invested capital} / (\text{WACC} - \text{Growth Rate}) \\
&= \$250{,}000 / (0.163 - 0.03) \\
&= \$250{,}000 / 0.133 \\
&= \$1{,}879{,}699
\end{aligned}
$$

Subtracting the book value of the debt, $300,000, from the estimated value of $1,879,699 implies a market value of the equity of $1,579,699. This results in capital structure weights of 16% for debt and 84% for equity. The calculated weights are significantly different from the book value weights of 30% for debt and 70% for equity that the analyst started with. Therefore, the analyst must adjust the weightings and recalculate using a second iteration.

Second Iteration

The calculated weights were lower for debt (15% vs. 30%) and higher for equity (85% vs. 70%) than the assumed weights. Using the first iteration as a guide, the analyst may adjust the capital structure weights to 20% for debt and 80% for equity. Including these amounts in the formula yields the following WACC calculation:

WACC = $(k_e \times W_e) + (k_{d/(pt)} [1\text{-}t] \times W_d)$
= $(0.22 \times 0.80) + (0.05 [1 - 0.40] \times 0.20)$
= $(0.176) + (0.03 \times 0.20)$
= $0.176 + 0.006$
= 0.182
= 18.2%

Proof – Second Iteration

Once again using an assumed growth rate of 3.0%, the proof of the second iteration is as follows:

Estimated value = Net cash flow to invested capital / (WACC – Growth Rate)
= $250,000 / (0.182 – 0.03)
= $250,000 / 0.152
= $1,644,737

The resulting calculated capital structure weights are:

Common equity	=	($1,644,737 - $300,000) / $1,644,737	=	81.7%
Long-term debt	=	$300,000 / $1,644,737	=	18.2%

Note that the calculated weights are much closer to the assumed weights than in the first iteration, 81.7% vs. 80% for equity and 18.2% vs. 20% for debt. This implies that a WACC of 18.2% is reasonable for this company.

Additional iterations may be performed in order to arrive at calculated weights that are even closer to the assumed weights.

Practice Pointer

The process of going through these iterative calculations is greatly simplified by use of automated spreadsheet functions such as the Iteration function in Excel or certain software programs that perform the iteration automatically.

Alternatively, here is an algebraic formula that bypasses the iterations:

$$E_{FMV} = \frac{NCF_{I/C} - D\,(C_D - g)}{C_E - g}$$

Legend:
E_{FMV} – Fair Market Value of Equity
$NCF_{I/C}$ – Net Cash Flow to Invested Capital
D – Total Interest Bearing Debt
C_D– After Tax Interest Rate
C_E – Cost of Equity
g – Long Term Sustainable Growth Rate

B. WACC – WHICH CAPITAL STRUCTURE TO USE

As noted earlier, use of the WACC can add versatility to the valuation in that it can be developed based on a number of assumptions involving the company's debt in its capital structure. These assumptions can include greater debt, less debt, or debt under different terms and may be based on the existing capital structure, a potential buyer's capital structure, an industry-average capital structure, or an optimal capital structure.

For example, if a controlling interest is being valued and the standard of value used is fair market value, the analyst can use an industry-average capital structure since a controlling interest would have the ability to change the capital structure of the company. On the other hand, if a non-controlling (minority) interest is being valued, the existing capital structure should be used as a non-controlling (minority) interest would not have the ability to change the existing capital structure.

If the analyst is valuing a controlling interest for a possible sale of the company and a potential buyer is known (investment value standard), then the potential buyer's capital structure or an optimal capital structure may be warranted for the calculation.

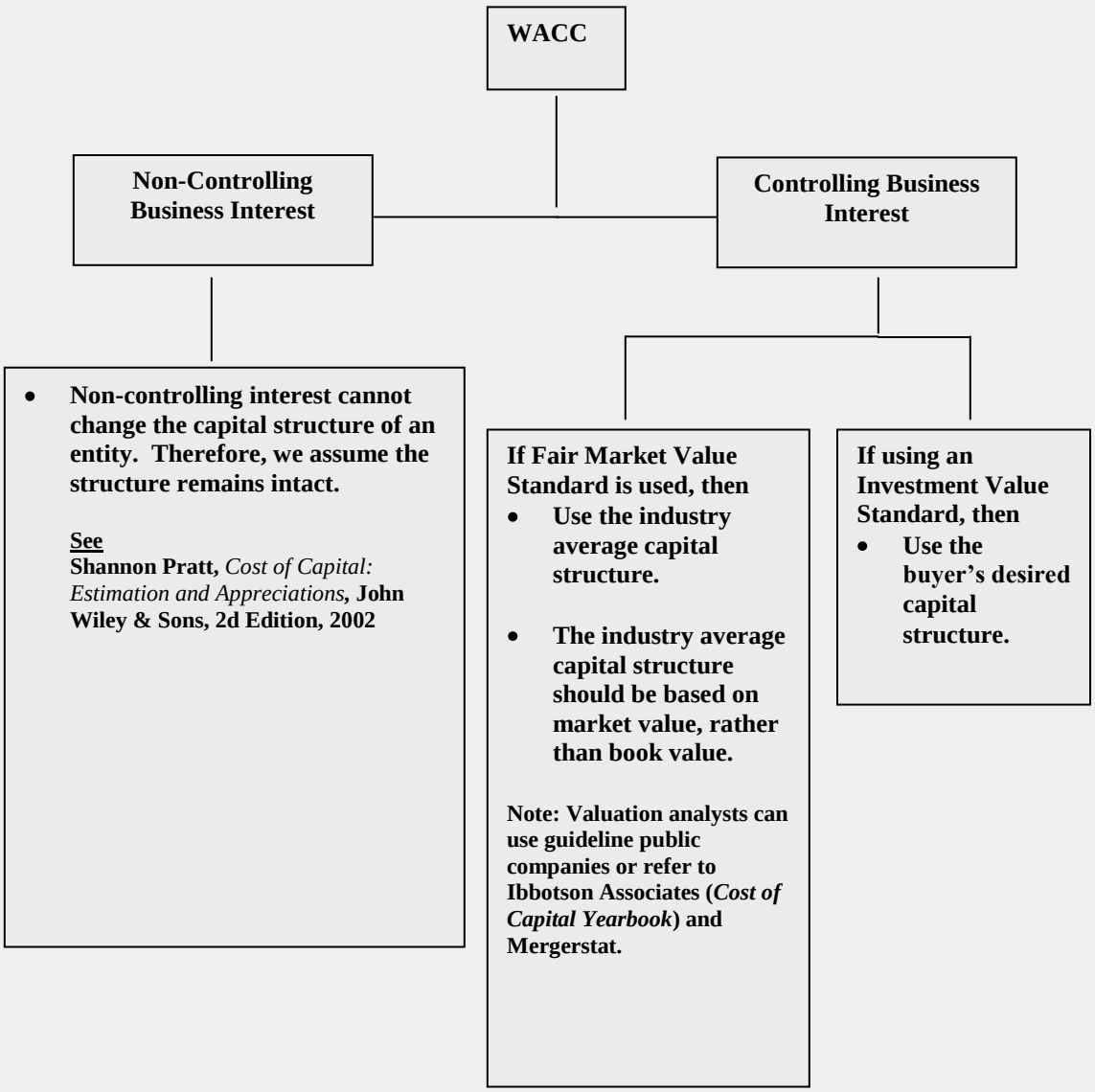
C. WACC – DETERMINING THE VALUE OF EQUITY

WACC is used primarily when the analyst is valuing the entire capital structure of a company (debt plus equity), and is applied to net cash flow to invested capital. WACC can still be used to value only the equity of a company. This is accomplished by calculating the value of the entire capital structure and then subtracting the company's debt, resulting in the value of the company's equity.

Practice Pointer

- **The Weighted Average Cost of Capital (WACC) used to value a closely held business may differ depending on whether non-controlling or controlling interest is being purchased.**

  The following flow chart provides an overview of current practice



VII. MARKET MULTIPLES

Market multiples can help an analyst compare a privately held company to the market, based on expectations the public has of similar publicly traded companies. The valuation analyst will need to determine which, if any, of the market-based multiples might apply to the subject company. Using market data, the valuation analyst calculates a number of ratios, such as P/E, P/CF, P/R, D/P and P/BV, and uses these same ratios, if applicable, to calculate the value of a share in a privately held company.

A. PRICE EARNINGS RATIO (P/E)

Price Earnings is probably the most commonly used market method to describe the price of a share of stock. This method utilizes price/earnings (P/E) ratios of comparable publicly traded companies involved in the same industry as the subject company. The rate is determined by calculating the weighted average of the inverse of the P/E ratios of publicly traded companies, possibly using multiple time periods.

Proponents of this method argue that the inverse or reciprocal P/E ratio of public companies in the same industry as the subject company is the best available comparable capitalization or discount rate for valuing a small, closely held business. P/E ratios are the inverse of the capitalization rate.

This method has some appeal due to the fact that P/E ratios for thousands of publicly traded companies are published daily.

The primary argument against this method is that large, diversified, publicly traded companies are not reasonably comparable to a smaller closely held business. Some factors behind this conclusion are:

1. Minority interests versus controlling interest
2. Capital structure
3. Stock market fluctuations
4. Supply versus demand for particular stocks
5. Diversity in reported financial information

P/E ratios are based on earnings after depreciation, amortization, interest on all debt, compensation to all employees (including stockholder/employees) and all federal and state corporate income tax. In order to use a P/E ratio, the analyst must be working with an earnings figure that is similar in all respects.

Using five-year average P/E ratios from five public companies, the rate is derived as follows:

Company 1	P/E	=	6.3
Company 2	P/E	=	7.6
Company 3	P/E	=	8.4
Company 4	P/E	=	5.2
Company 5	P/E	=	5.9
Total			33.4
Average	(33.4 /5)		6.68

Discount or Capitalization Rate (after-tax) $= \frac{1}{6.68} = 14.97\%$

Use of publicly-traded company market multiples (also known as Guideline Public Company data) to estimate private-company market enterprise value should be done with caution. Numerous courts have ruled that public companies are so materially different from private companies that no comparison is possible. Frequently, the valuation analyst will discuss guideline public company data in their private company valuation reports but will only use the data as a sanity check.

B. PRICE/CASH FLOW (P/CF)

Price per share divided by cash flow.

1. Cash flow is typically defined, for purposes of this calculation, to be net income plus depreciation and amortization.
2. This measure is considered relevant for companies with high non-cash charges reflected in the income statement—usually found in depreciation and amortization.

C. PRICE/REVENUE (P/R)

Price per share divided by the revenue.

1. This multiple works well for service type companies, or those with few assets. These kinds of companies will often sell at prices related to their revenues.
2. The assumption behind this ratio is that a certain level of revenue will generate a certain "level" of earnings, or earnings potential. The higher the return on revenue (earnings divided by revenue) the higher the price to revenue will be.
3. A regression analysis can often be fit nicely to this market multiple.

D. DIVIDEND/PRICE (D/P)

Dividend divided by price is usually called the dividend yield.

1. Most closely held companies don't pay dividends due to the double taxation, making this approach to pricing a share of closely held stock very difficult.
2. Some public stocks do not sell well based on dividend yield, as the companies pay minimal dividends or none at all. Others, such as REITS, pay a high proportion of earnings as dividends and will have a correspondingly high yield. In either case, the decision to pay or not pay a dividend is not influenced by any minority owner so the approach is likely not relevant when one is valuing a minority interest.

3. Companies, public or private, that do not pay dividends often may actually have the capacity to pay a dividend, which can be calculated. If the analyst can show that such a payout would not appreciably deny the company its ability to finance operations and growth, the price to dividend ratio might be applicable.

E. PRICE/BOOK (P/BV)

The market price per share divided by book value per share.

1. Book value, or common equity, per share is total owners' equity minus preferred stock divided by the number of common shares outstanding.
2. The purpose of this ratio is to test whether the market price is worth more (or less) than the cost of the assets. If the result is greater than one, it indicates market value exceeds book value and can often be used as a sign of competent management.

F. EARNINGS PER SHARE (EPS)

EPS is net income minus preferred stock dividends divided by the number of common shares outstanding.

1. A trailing EPS is calculated for the past year.
2. The valuation analyst must also decide whether to make this calculation based on fully diluted earnings or primary (undiluted) earnings.

VIII. LESS FREQUENTLY USED SMALL BUSINESS BUILD-UP MODELS

The analyst's main objective in developing a discount or capitalization rate is the assessment of an appropriate risk premium. Therefore, a method which analyzes many different components of risk could be the most theoretically sound. Many approaches for this method exist to determine an appropriate capitalization or discount rate.

A business build-up model presents the valuation analyst with an alternative for computing a capitalization or discount rate when the subject company is materially smaller than those companies used to derive cost of capital estimates by Ibbotson. Instead of relying upon market data from large publicly traded entities, this approach requires the valuation analyst to inspect the small business from various aspects of risk in order to conclude an appropriate risk premium for the enterprise.

The theoretical basis for this approach is the same as that of the capital asset pricing model (CAPM), which is: "Investors in 'risky' investments require a higher rate of return, above and beyond a risk free or safe investment rate, as compensation for bearing the risk associated with holding the investment."

A. SCHILT'S RISK PREMIUM GUIDELINES

Schilt's Risk Premium for Discounting Projected Income Streams

Category	Description	Risk Premium
1.	Established businesses with a strong trade position, well financed, with depth in management, whose past earnings have been stable and whose future is highly predictable.	6 - 10%
2.	Established businesses in a more competitive industry that are well financed, have depth in management, have stable past earnings and whose future is fairly predictable.	11 - 15%
3.	Businesses in a highly competitive industry that require little capital to enter, no management depth, a high element of risk and whose past record may be good.	16 - 20%
4.	Small businesses that depend upon the special skill of one or two people. Larger established businesses that are highly cyclical in nature. In both cases, future earnings may be expected to deviate widely from projections.	21 - 25%
5.	Small "one person" businesses of a personal services nature, in which the transferability of the income stream is in question.	26 - 30%

Note: "The risk premium chosen is added to the risk-free rate...." The resulting figure is the risk-adjusted capitalization rate for use in discounting the projected income stream. Because of the wide variation in the effective tax rates among companies, these pre-tax figures are designed to be used with pre-tax income.

Source: James H. Schilt, "Selection of Capitalization Rate – Revisited" Business Valuation Review, June 1991, p. 51.

1. Description

a) It is a simple method.
b) It is somewhat reasonable.
c) It does not identify the risk premium for each of the possible risk factors.
d) Ranges require subjective conclusions.

B. THE RISK RATE COMPONENT MODEL (RRCM)

The RRCM[11] is a business build-up model designed to identify an appropriate capitalization rate based on the perceived risks associated with an enterprise. Many valuation analysts believe that a business build-up model is a better approach to use when the enterprise is considered too small for market data methods.

The RRCM begins by taking a safe or reasonable rate of return (e.g., intermediate term bond rate) and adds to that rate a weighted average risk premium for each of the following general risk factor categories:

[11] Available in BVMPro.

Primary Factors

1. Competition
2. Financial strength
3. Management ability and depth
4. Profitability and stability of earnings

Other Factors to Consider

1. National economic effects
2. Local economic effects

The Risk Rate Component Model identifies more specific risk factors that fall within the four primary risk factor categories. Each of these specific risk factors is evaluated and assigned a risk premium percentage and then weighted according to the relative degree of influence it has on the general category where it resides. Then a weighted average of all of the specific risk factors for each category is calculated. These weighted averages then become the risk premium factors for each of the general risk factor categories. The general risk categories can then be weighted relative to the perceived importance that each general category has relative to the others.

Section 5 of Revenue Ruling 59-60 requires the valuation analyst to use informed judgment when weighing the various factors or components. Necessarily the valuation analyst using the RRCM should document in working papers how each component has been considered. Valuation analysts can reduce the subjective nature of the analysis of the various components by conducting site visits, gathering industry information, conducting interviews with management and other informed persons and performing detailed analytical analysis through ratio analysis. Risk can be quantified in several ways: as weak, no effect, or strong; or High, Medium, Low and No Risk, or; Heavy, Moderate, Light, None. The valuation analyst setting up a quantification chart should be consistent in his or her application.

Each risk factor that can be analyzed in ratio analysis should, where possible, be compared to similar ratios from industry publications (e.g., RMA, etc.) in order to compare the position/performance of the subject company to comparable companies.

1. RRCM Risk Factors

The four general risk factor categories: Competition, Financial Strength, Management Ability and Depth and Profitability and Stability of Earnings are synthesized from the Black/Green Build-up Summation Method, the James Schilt Risk Premium Guidelines, The Complete Guide to Buying a Business by Arnold Goldstein (1983), How to Value a Small Business, Real Estate Today, by Harold S. Olafson (1984), Selling Your Business, Business Week, Bradley Hitchings (1985) and the BNA Tax Management: Estates, Gifts and Trusts Portfolios (221d) (1985).

The following table lists suggested underlying risk components the analyst should review for each category: Each risk component can be analyzed by ratio analysis [R], questionnaires to be completed with management [Q] or through other analysis and worksheets [A] [12].

[12] Suggested questionnaires and analytical worksheets can be found in The Value of Risk© 2001 and 2002, Hanlin and Claywell.

RRCM Risk Factors

Competition		**Financial Strength**	
Q	Proprietary content (including patents and copyrights)	R	Current ratio
A	Relative size of company	R	Quick ratio
Q	Relative product or service quality	R	Sales to working capital ratio
Q	Product or service differentiation	R	Accounts receivable to working capital ratio
Q	Covenant not to compete	R	Inventory to working capital ratio
Q	Market strength – competition	R	Net sales to inventory turnover
A	Market size and share	R	Total sales to assets
Q	Pricing competition	R	Net fixed assets to net worth
Q	Ease of market entry	R	Miscellaneous assets to net worth
	Other pertinent factors specific to the subject company	R	Total debt to net worth
		R	Total assets to total equity
Management Ability and Depth		R	Total debt to assets
R	Accounts receivable turnover	R	Long-term debt to equity
R	Accounts payable turnover	R	Interest coverage
R	Inventory turnover		Other pertinent factors specific to the subject company
R	Fixed asset turnover		
R	Total asset turnover	**Profitability and Stability of Earnings**	
R	Employee turnover	Q	Years in business
R	Management depth	Q	Industry life cycle
Q	Facilities condition	R	Return on sales (before taxes)
Q	Family involvement	R	Return on assets
Q	Books and records – quality and history	R	Return on equity
Q	Contracts for sales	R	Operating earnings growth rate
Q	Contracts for purchases	R	Sales growth rate
Q	Contracts for management	R	Trading ratio (sales to net worth)
Q	Contracts – other	R	Standard Deviation
R	Gross margin		Other pertinent factors specific to the subject company
R	Operating margin		
R	Operating cycle		
	Other pertinent factors specific to the subject company		

The Risk Rate Component Model assumes that the risk premiums and the safe rate of return are on a pre-tax basis; therefore, this method generates a capitalization rate for use on a pre-tax basis. If the valuation analyst using the RRCM desires a discount rate, then a factor for long-term growth should be added.

The business build-up summation table, below, shows how the RRCM can work.

BUILD-UP SUMMATION TABLE

Risk Factors	(1) Risk Indicator[13]	(2) Weight	(3) WEIGHTED RISK INDICATOR
Competition:			
Proprietary Content (including Patents & Copyrights)	6.0	1.00	6.0
Relative Size of Company	2.0	1.00	2.0
Relative Product or Service Quality	4.0	1.00	4.0
Product/Service Differentiation	4.0	1.00	4.0
Covenant not to compete	0.0	0.00	0.0
Market Strength – Competition	2.0	1.00	2.0
Market Size and Share	2.0	1.00	2.0
Pricing Competition	6.0	1.00	6.0
Ease of Market Entry	2.0	1.00	2.0
	8.0	1.00	8.0
Total Weight Factors		**9.00**	**36.0**
Total Weighted Average			**4.0%**
Financial Strength:			
Current Ratio	4.0	1.00	4.0
Quick Ratio	4.0	1.00	4.0
Sales to Working Capital	4.0	1.00	4.0
Accounts Receivable to Working Capital	4.0	1.00	4.0
Inventory to Working Capital	6.0	1.00	6.0
Net Sales to Inventory turnover	7.0	1.00	7.0
Total Assets to Sales	5.0	1.00	5.0
Net Fixed Assets to Net Worth	3.0	1.00	3.0
Miscellaneous Assets to Net Worth	0.0	1.00	0.0
Total Debt to Net Worth	8.0	1.00	8.0
Total Assets to Total Equity	7.0	1.00	7.0
Total Debt to Assets	8.0	1.00	8.0
Long–term Debt to Equity	7.0	1.00	7.0
Interest Coverage	4.0	1.00	4.0
Total Weight Factors		**14.00**	**71.0**
Total Weighted Average			**5.07%**
Management Ability and Depth:			
Accounts Receivable Turnover	2.0	1.00	2.0
Accounts Payable Turnover	8.0	1.00	8.0
Inventory Turnover	2.0	1.00	2.0
Fixed Asset Turnover	4.0	1.00	4.0
Total Asset Turnover	4.0	1.00	4.0
Employee Turnover	8.0	1.00	8.0
Management Depth	8.0	1.00	8.0
Facilities Condition	6.0	1.00	6.0
Family Involvement	8.0	1.00	8.0
Books & Record – Quality & History	6.0	1.00	6.0
Contracts	8.0	1.00	8.0
Gross Margin	4.0	1.00	4.0
Operating Margin	6.0	1.00	6.0
Total Weight Factors		**13.00**	**74.0**
Total Weighted Average			**5.69%**
Profitability & Stability Of Earnings:			
Years In Business	2.0	1.00	2.0
Industry Life Cycle	4.0	1.00	4.0
Return On Sales (before taxes)	6.0	1.00	6.0
Return On Assets	4.0	1.00	4.0
Return On Equity	4.0	1.00	4.0
Operating Earnings Growth Rate	5.0	1.00	5.0
Trading Ratio (Sales/Net Worth)	5.0	1.00	5.0
Earnings Standard Deviation	6.0	1.00	6.0
Total Weight Factors		**8.00**	**36.0**
Total Weighted Average			**4.50**
TOTAL RISK PREMIUM FACTOR			**19.26%**

[13] A High risk would receive a score value of 10.0; a Medium High risk 7.5; Medium risk 5.0; Medium Low risk 2.5; Low risk 1.0; No risk 0.0. See *The Value of Risk* ©, Hanlin & Claywell, 2002, p. 23.

2. SUMMARY OF RRCM SUMMATION METHOD

Risk Factor (By General Category):

Competition	4.00%
Financial Strength	5.07%
Management Ability and Depth	5.69%
Profitability and Stability of Earnings	4.50%
Total Weighted Average Risk Factor Premiums:	**19.26%**

Calculation of Capitalization Rate:	
Total Weighted Average Risk Factor Premium	19.26%
Assumed Safe (Reasonable) Rate of Return	5.50%
National Economic Premium (or Discount)	1.00%
Local Economic Premium (or Discount)	1.00%
Indicated Capitalization Rate	**26.76%**
Rounded capitalization rate	27.00%

C. THE JEFF JONES METHOD – DEVELOPMENT OF A PRICING MULTIPLE

"Most business brokers who use the multiple of discretionary earnings method have some kind of a worksheet listing the factors that affect the multiple. *(The multiple is an inverted capitalization rate, where a 20 percent capitalization rate infers a multiple of five (100%/20%).* In addition, there is a rating and weighting system for each factor. The factors and rating and weighting scheme may vary considerably from one broker to another. This variability depends to a great extent on the types of businesses the brokers specialize in or typically tend to sell."[14]

"There is general recognition that the factors and the ratings and weightings are quite subjective. However, brokers who have the experience of selling certain types of businesses several times a year tend to develop a feel for how buyers perceive those businesses in the particular market at the particular time. This ongoing connection with the transactional market helps the brokers use their respective analytical systems in advising sellers and buyers about prices that the market is likely to accept."[14]

"The most publicized such analytical framework is that developed by Certified Business Brokers. It is generalized to apply to most types of small businesses typically selling in the $50,000 to $500,000 range. The Certified Business Brokers Appraiser's Analysis Table is shown below. The multiple developed is applied to discretionary earnings.[15] This would produce an indicated value for the intangible and operating assets, which include furniture, fixtures and equipment and inventory at a normalized amount. If any other elements of working capital or other assets were included in the sale, or any liabilities assumed, the indicated value would be adjusted accordingly."[14]

[14] Quoted: *Valuing Small Businesses & Professional Practices-3rd Edition*, Pratt, Reilly & Schweihs, McGraw-Hill, 1998 p. 32-333 and Handbook of Business Valuations, West and Jones, Wiley & Sons, 1992.

[15] The International Business Brokers Association defines discretionary earnings as the earnings of a business enterprise prior to income taxes, non-operating income and expense, nonrecurring income and expense, depreciation and amortization, interest income and expense, and owner's total compensation for those services that could be provided by a sole owner/manager. See *Valuing Small Businesses & Professional Practices – 3rd Edition*, Pratt, Reilly & Schweihs, McGraw-Hill, 1998, p. 329.

APPRAISER'S ANALYSIS TABLE

Rating Scale	Description	Selected Multiple	Weight	Weighted Value
Historical Profits				
0.1– 1.0	Negative to break even			
1.1–2.0	Positive, but below industry norm	1.25	10	12.50
2.1–3.0	Industry norm or above			
Income Risk				
0.1–1.0	Continuity of income at risk			
1.1–2.0	Steady income likely – three to five years	1.00	9	9.00
2.1–3.0	Profitability assured – five plus years			
Terms of Sale				
0.1–1.0	Seller requires all cash			
1.1–2.0	Reasonable terms available	1.50	8	12.00
2.1–3.0	Exceptional terms available			
Business Type				
0.1–1.0	Service business with few assets			
1.1–2.0	Equip and/or inventory are significant component of total value	1.50	7	10.50
2.1–3.0	High cost of entry, equip and/or inventory are major component of total value			
Business Growth				
0.1–1.0	Declining and further decline likely			
1.1–2.0	Flat or at inflationary levels	.50	6	3.00
2.1–3.0	Rapid growth withy more expected			
Location/Facilities				
0.1–1.0	Less than desirable to tolerable			
1.1–2.0	Acceptable to average	2.50	5	12.50
2.1–3.0	Above average to superior			
Marketability				
0.1–1.0	Limited market – special skills required			
1.1–2.0	Normal market – needed skills available	1.00	4	4.00
2.1–3.0	Large market – many qualified buyers			
Desirability				
0.1–1.0	No status, rough or dirty work			
1.1–2.0	Respectable and satisfactory	1.00	3	3.00
2.1–3.0	Challenging and attractive environment			
Competition				
0.1–1.0	Highly competitive and unstable market			
1.1–2.0	Normal competitive conditions	2.00	2	4.00
2.1–3.0	Little competition/high startup cost			
Industry				
0.1–1.0	Declining and further decline likely			
1.1–2.0	Flat or at inflationary levels	2.00	1	2.00
2.1–3.0	Rapid growth with more expected			

SUMMARY OF APPRAISER'S ANALYSIS TABLE

Categories	Selected Multiple	Weight	Weighted Values
Historical Profits	1.25	10	12.50
Income Risk	1.00	9	9.00
Terms of Sale	1.50	8	12.00
Business Type	1.50	7	10.50
Business Growth	0.50	6	3.00
Location/Facilities	2.50	5	12.50
Marketability	1.00	4	4.00
Desirability	1.00	3	3.00
Competition	2.00	2	4.00
Industry	2.00	1	2.00
Total		55	72.50
Selected Multiple			1.32

For each of the 10 risk characteristics, a multiple is selected and a weight assigned, so that the weighted multiple comes out between zero and three. This multiplier is applied to "Discretionary Earnings."

Net working capital and the value of non-operating or excess assets are added to the above-capitalized value of discretionary earnings and long-term debt (including current portions) is subtracted. If valuing a partial interest, the pro-rata portion of this result normally would be reduced by a minority interest discount (if the standard of value is fair market value).

The multiple of discretionary earnings method clearly produces an indicated value for a controlling owner. Further, the model was developed to aid in pricing an entire business for sale. There have been no mechanisms developed to estimate a minority ownership interest.

D. BLACK/GREEN BUILD-UP SUMMATION METHOD

This method, introduced in 1991, begins by taking a safe or reasonable rate of return (e.g., intermediate term Treasury bond rate) and adds to that rate a weighted average risk premium for each of the following general risk factor categories:

1. Competition
2. Financial strength
3. Management ability and depth
4. Profitability and stability of earnings
5. National economic effects
6. Local economic effects

This method formed the basis for several build-up models, including RRCM and Value-Netex. The Black/Green Method assumes that the risk premiums and the safe rate of return are on a pre-tax basis; therefore, this method generates a capitalization rate for use on a pre-tax basis. This rate can be converted to a discount rate by adjusting for growth.

At the time of its inception, this model relied upon the analyst's experience and intuition to assign a value to each risk component. Many valuation analysts came to the conclusion that the Black/Green Method was too subjective and lacking in detailed support for the value of each of the risk components.

E. VALUE-NETEX

The Value-Netex model is derived directly from the Black/Green model. This model is used in the software program eValPro™ (no longer available) developed by Value-Netex Corporation. The following is a brief description of the model:

The Value-Netex Build-Up Method is a new method that effectively enhances or builds upon the Black/Green Build-Up Summation Method. Robert L. Green, CPA, CVA, CFE, CM&AA, was co-developer of the Black/Green Build-Up Summation Method in 1991. It is a method that was largely based on the same factors that are included to derive a discount rate under the CAPM (that being the Safe Rate of Return and a factor for the Risk Premium in excess of the safe rate with a beta of one assumption). The theoretical basis for this method is that investors investing in investments that have more risk than government bonds require a higher rate of return for the increased level of risk above the risk-free rate. The principle differences between the Value-Netex Build-Up Method and the Black/Green Build-Up Method, is that the Value-Netex Method defines areas of risk differently, which in its authors opinion allows the analyst the ability to take into consideration more issues when developing the rate. The Value-Netex Method considers the following general risk factor categories, broken down between Quantitative categories and Qualitative categories; the range of rates is also included.

Quantitative Categories of Risk:

	High Risk	Med/ High Risk	Med Risk	Med/ Low Risk	Low Risk
Liquidity	10	8	6	4	2
Leverage	10	8	6	4	2
Operations	10	8	6	4	2
Cost Control	10	8	6	4	2
Growth	10	8	6	4	2

Qualitative Categories of Risk:

	High Risk	Med/ High Risk	Med Risk	Med/ Low Risk	Low Risk
Competition	10	8	6	4	2
Management	10	8	6	4	2
Stability	10	8	6	4	2

Each of the general categories contains numerous underlying and various specific risk factors. When assessed by the valuator, a risk rate is applied to the specific risk factor. The risk rates that are applied are dependent upon the valuator's informed judgment as to the specific attributes of the subject company. The risk rates applied to the specific factors can be weighted to take into consideration that some specific risk factors may have greater significance than the other specific risk factors contained in each category. In addition, the total weighted risk of each category can also be weighted according to the relative significance that a general category of risk would have to the other general categories. This is a somewhat subjective process, however, valuation is not an exact science and the valuation process requires that the analyst

maintain objectivity, applies informed knowledge and is reasonable in his or her approach. These concepts, discussed in Revenue Ruling 59-60, are the basis and support for the theory and application of the Value-Netex and Black/Green Methods.

Quantitative Risk Categories:

Liquidity	*3.00%*
Leverage	*4.00%*
Operations	*3.00%*
Cost Control	*4.00%*
Growth	*3.00%*
Total Quantitative Risk	*17.00%*

Qualitative Risk Categories:

Competition	*6.00%*
Management	*5.00%*
Stability	*4.00%*
Total Qualitative Risk	*15.00%*
Specific Company Risk Premium	***32.00%***

Calculation:

Risk-Free Rate	*5.98%*
Add: Specific Company Risk Premium	*32.00%*
Total Pre-tax Capitalization Rate	***37.98%***

—— Description provided by Robert L Green, CPA, CFE, CVA, CM&AA, March 21, 2003.

IX. RATES DEVELOPED FROM VALUE DERIVED MODELS

A. ECONOMIC VALUE ADDED (EVA) – STERN STEWART GROUP

Economic Value Added (EVA) is a residual income measure that subtracts all of the cost of capital from the operating profits generated in the business. The simple formula to calculate this value is:

$EVA = (r – c) \times k$

Where r represents the rate of return (net operating profits after-taxes (NOPAT), divided by capital), c represents the cost of capital and k is the economic book value of the capital committed to the business. For example, if r is 20 percent (NOPAT is $20,000), c is 12.5 percent, capital is $100,000 and then the value added is $7,500. The rate of return can be calculated for this formula by dividing net operating profit after-tax by the capital committed to the business.

Although in any given business there are countless individual things that people can do to create value, eventually they all must fall into one of the three categories measured by an increase in EVA. EVA will rise if operating efficiency is enhanced, if value-adding new investments are

undertaken and if capital is withdrawn from uneconomic activities. Specifically, EVA increases when:

1. The rate of return earned on the existing base of capital improves; that is, more operating profits are generated without tying up more funds in the business.
2. Additional capital is invested in projects that return more than the cost of obtaining the new capital.
3. Capital is liquidated from, or further investment is curtailed in substandard operations where inadequate returns are being earned.

The EVA model is used particularly by companies to manage the allocation of resources for maximizing the market value to shareholders. EVA rejects as primary management tools such measures as rate of return, earnings per share, earnings growth and even cash flow by injecting into the equation the capital resource factor required to sustain earnings, the need for new capital resources and the return of capital resources for reinvestment or distribution to shareholders.

Using the initial example from above, but assuming that the enterprise must allocate an additional $15,000 to capital in order to sustain current earnings, the EVA is computed to be $5,625.

Although EVA yields the same answer for a given forecast that discounting cash flow yields, the EVA approach has the advantage of showing how much value is being added as a result of the capital employed in each year of the forecast. Also, it is the only method that can clearly connect prospective capital budgeting and strategic investment decisions with the way in which actual operating performance could subsequently be evaluated.

There is a great deal more to this method. For details, see *The Quest for Value, A Guide for Senior Managers*, by G. Bennett Stewart, III, Harper Collins Publishers Inc., 1991, 1999.

B. SHAREHOLDER VALUE ADDED (SVA) – ALFRED RAPPAPORT

Corporate boards and CEOs almost universally embrace the idea of maximizing shareholder value. It has become politically correct, though it is not always fully implemented in practice. Where before the 1990s shareholder value applications consisted principally of evaluating capital expenditures and pricing acquisitions with discounted cash-flow methods, companies now incorporate shareholder value measurements into planning and evaluating the overall performance of their business.

What has not changed is the fundamental shareholder value model itself. It continues to reflect the way rational participants in a market-based economy assess the value of an asset—the cash it can be expected to generate over time, adjusted for the risks of that cash stream.

The shareholder value approach estimates the economic value of an investment by discounting forecasted cash flows by the cost of capital. These cash flows, in turn, serve as the foundation for shareholder return from dividends and share-price appreciation. The basic valuation parameters or value drivers—sales growth rate, operating profit margin, income tax rate, working capital investment, fixed capital investment, cost of capital, and forecast duration—are developed and incorporated in shareholder valuation calculations.

The risk of the investment is no greater than the company's existing overall risk. Terminal value is calculated based upon cash flow. There is no unique formula for the terminal or

residual value, although Rappaport suggests that the analysis should consider three specific scenarios for this portion of the computed value. The SVA models consider a terminal value with no growth, some growth or with consideration to liquidation of the enterprise.

SVA also addresses the change in value or the amount created by the forecasting scenario, which results from corporate investment at rates in excess of the cost of capital.

For details, including the SVA Network, see Creating Shareholder Value, New York: The Free Press (Simon & Schuster, Inc.), by Rappaport, 1986, 1998.

X. COMMON ERROR

The following illustration demonstrates the impact when the analyst is not consistent with the application of a single method when performing a valuation.

Assumptions:

Weighted average economic earnings (pre-tax) = $157,500

Appropriate capitalization rate (pre-tax) = 30%; after-tax = 18%

Income Tax Rate = 40%

Pre-Tax	**After-Tax**
$157,500 (Pre-tax)	$157,500 (Pre-tax)
30% (Pre-tax)	18% (After-tax)
Correct Indicated Value = $525,000	Incorrect Indicated Value = $875,000

$350,000
Difference!

XI. CONVERTING PRE-TAX RATES TO AFTER-TAX RATES

A. OBSERVATION

It would obviously be an error to apply pre-tax capitalization or discount rates to after-tax earnings and after-tax capitalization or discount rates to pre-tax earnings. Therefore, the analyst may find it necessary to convert rates.

B. CONVERT PRE-TAX CAPITALIZATION RATES TO AFTER-TAX RATES

Formula:

After-tax cap rate = Pre-tax cap rate x (1 – Tax rate)

Example:

Assume:

Pre-tax capitalization rate	=	30%
Tax rate	=	40%

Calculate after-tax rate:

After-tax rate	=	30% x (1 - .40)
After-tax capitalization rate	=	18%

C. CONVERT AFTER-TAX CAPITALIZATION RATES TO PRE-TAX RATES

Formula:

Pre-tax capitalization rate = After-tax cap rate ÷ (1 – Tax rate)

Example:

Assume:

After-tax capitalization rate	=	18%
Tax rate	=	40%

Calculate after-tax rate:

Pre-tax rate	=	18%÷ (1 - .40)
Pre-tax capitalization rate	=	30%

Recommended reading includes, but is not limited to:

- Abrams, Jay, *Quantitative Business Valuation, A Mathematical Approach for Today's Professionals,* Chapter 2 (Using Regression Analysis), and Chapter 3 (Annuity Discount Factors and the Gordon Model).
- Blackman, Irving L., *Valuing Your Privately Held Business, The Art & Science of Establishing Your Company's Worth*, Chapter 11 & 12 (Recapitalizations).
- Brown, Marvin T., Executive Compensation and the Cost of Capital, *The Valuation Examiner*, J/A 2003.
- Claywell, J. Richard, Quantifying Financial Risk in Capitalization Rates, *The Valuation Examiner*, 2qtr, 1994.
- Copeland, Tom, et. al., *Valuation Measuring and Managing the Value of Companies*, Chapter 8 (Estimating the Cost of Capital).
- Damodaran, Aswath, Damodaran on *Valuation, Security Analysis for Investment and Corporate Finance*, Chapter 5 (Estimation of Growth Rates).
- Elmaleh, Andrew, Do Investors Demand Higher Rates of Return on Risky Investments in Closely Held Businesses?, *The Valuation Examiner*, M/J 2003.
- Elmaleh, Michael S., The Subjectivity and Relativity of Risk Assessments in Investment Decisions, *The Valuation Examiner*, S/O 2003.
- Gulden, Allen J., Are We Understanding Unsystematic Risk Premium?, *The Valuation Examiner*, N/D 1001.
- Helfert, Erich, *Techniques of Financial Analysis*, Chapter 8 (Cost of Capital and Business Decisions).
- Hitchner, James R., *Financial Valuation Applications and Models*, Chapter 4 (Income Approach, Part 11, Determination of The Future Benefit Stream (Cash Flows) and 12 (Defining the Benefit Stream) and 13 (Defining Net Cash Flow), Chapter 5 (Cost of Capital/Rates of Return).
- Ibbotson Associates, *Stocks, Bonds, Bills and Inflation, Valuation Edition 2005 Yearbook*, Chapter 2 (Introduction to the Cost of Capital), Chapter 3 (The Buildup Method), Chapter 4 (Overview of Cost of Equity Capital Methods), and Chapter 6 (Beta Estimation Methodologies).
- Kasper, Larry, Business *Valuations Advanced Topics*, Chapter 7 (The Capital Asset Pricing Model), Chapter 7 (Estimating Betas) and Chapter 10 (Adjusting Betas).
- Lebbon, Tim, Cost of Capital, Converting Real to Nominal, *The Valuation Examiner*, N/D 2000.
- Moran, E. F. Jr., The Marketability Build-Up Methods, *The Valuation Examiner*, S/O 2001.
- Murray, Andrew, Side-Stepping the Internal Inconsistency Imbedded in the Cash Flow/WACC Method of Capitalizing Future Benefits, or Fun With Algebra, *The Valuation Examiner*, S/O 2003.
- Pratt, Shannon P., *Cost of Capital Estimation and Applications*, Chapter 4 (Discounting versus Capitalizing), Chapter 7 (Weighted Average Cost of Capital), Chapter 9 (Capital Asset Pricing Model) and Chapter 10 (Proper Use of Betas).
- Pratt, Shannon P., R. F. Reilly and R. P. Schweihs, *Valuing a Business, The Analysis and Appraisal of Closely Held Companies*, Part III (Business Valuation Approaches and Methods).
- Schilt, James H., Selection of Capitalization Rates for Valuing a Closely-Held Business, *The Valuation Examiner*, J/J 1996.

BUSINESS VALUATIONS:

FUNDAMENTALS, TECHNIQUES AND THEORY (FT&T)

CHAPTER 5

REVIEW QUESTIONS

FT&T

CHAPTER REVIEW QUESTIONS

Chapter 5: Capitalization/Discount Rates

1. What is a capitalization rate?

   a. The calculated external factor and internal factor multiplied by the investment factor
   b. Divisor (or multiplier) used to convert a defined stream of income to present value
   c. The price/earnings ratio divided by the dividend paying capacity
   d. Rate of return used to convert a series of future income amounts to their present value

2. What is a discount rate?

   a. The calculated external factor and internal factor multiplied by the investment factor
   b. Divisor or multiplier used to convert a defined benefit stream to present value
   c. The price/earnings ratio divided by the dividend paying capacity
   d. A rate of return used to convert a series of future income amounts to their present value

3. Earnings for Jasper Company for the last five years are shown below. What are the weighted average historical earnings?

Year	Earnings	Weight
1999	1,230,000	1
2000	1,240,000	2
2001	1,245,000	3
2002	1,230,000	4
2003	1,230,000	5

   a. 1,230,000
   b. 1,234,333
   c. 3,703,000
   d. 7,714,581

4. Using the weighted average historical earnings from question #3, if the calculated discount rate is 15% and long-term growth is 3%, what is the indicated value of Jasper Company based on a capitalization of single=period earnings method?

   a. $ 8,228,900
   b. $15,429,200
   c. $10,286,100
   d. $10,594,700

5. A capitalization rate and a discount rate are essentially the same thing.

    a. True
    b. False

6. The price earnings ratios for five public companies are: 8.20, 4.60, 5.00, 4.86, and 2.10. The after-tax capitalization rate is:

    a. 16.00%
    b. 18.08%
    c. 20.19%
    d. 24.76%

7. The primary formula for the Capital Asset Pricing Model (CAPM) is:

    a. Expected return = risk-free rate divided by beta multiplied by the expected return on a market portfolio
    b. Expected return = risk-free rate multiplied by beta multiplied by the expected return on a market portfolio less the risk-free rate.
    c. Expected return = risk-free rate plus beta multiplied by the expected return on a market portfolio less the risk-free rate.
    d. Expected return = beta divided by the risk-free rate multiplied by the expected return on a market portfolio less the risk-free rate

8. To calculate the weighted average cost of capital (WACC):

    a. Calculate the cost of debt plus the cost of equity in proportion to their book values
    b. Calculate the weighted average earnings and divide by the ratio of debt to equity
    c. Calculate the after-tax weighted cost of debt and add the weighted cost of equity
    d. Calculate the interest rate on a mid-range treasury bond and divide by beta

9. An estimate of a long-term sustainable growth rate should:

    a. Equal inflation plus the real volume growth that can be achieved with additional capital investment
    b. Equal inflation less the real volume growth that can be achieved with additional capital investment
    c. Equal inflation plus the real volume of growth that can be achieved without additional capital. Investment
    d. None of the above

10. Earnings per share is:

    a. The price of risk less the difference between the expected rate of return on a portfolio and the reasonable rate
    b. The price of the dividend divided by the price
    c. The market price per share divided by the book value per share
    d. The net income less preferred stock dividends divided by the number of common shares outstanding

11. To convert a pre-tax capitalization rate to after-tax capitalization rate:

    a. Multiply the pre-tax capitalization rate by 1 minus the expected tax rate
    b. Divide the after-tax capitalization rate by 1 minus the expected tax rate
    c. Multiply the pre-tax capitalization rate by 1 plus the expected tax rate
    d. Divide the after-tax rate by 1 plus the expected tax rate

12. Practice Exercise:

    Match the appropriate discount or capitalization rate to the benefit stream.

Benefit Stream	***Discount/Capitalization Rate***
Capitalization of Earnings/Cash Flow	*Ibbotson Build-Up Approach*
	+ Risk-free rate
	+Equity risk premium
Pre-tax earnings (income before taxes)	*+Size premium*
	+ Company specific risks
After-tax earnings (net income)	*= After-tax net cash flow discount rate*
	– Long-term sustainable growth rate
Net cash flow to invested capital	*= After-tax net cash flow capitalization rate for next year*
	÷ Adjustment for current year
Net cash flow to equity	*= After-tax net cash flow capitalization rate for current year*
	+ Cash to earnings factor
Pre-tax excess earnings	*= After-tax net income capitalization rate for current year*
	+ Intangible earnings factor
After-tax excess earnings	*= After-tax intangible capitalization rate for the current year*
	Tax effect
Discounting Future Cash Flows	*= Pre-tax net income capitalization rate for current year*
	= Pre-tax intangible capitalization rate for the current year
Projected cash flows	
	Weighted Average Cost of Capital (WACC)
	+ Weighted Cost of Debt
	+ Weighted Cost of Equity
	= WACC

13. General expectations of the particular business being valued, the size of the business being valued, and the nature of the business being valued are examples of:

    a. External factors that may influence the capitalization or discount rate
    b. Internal factors that may influence the capitalization or discount rate
    c. Investment factors that may influence the capitalization or discount rate
    d. Marketability factors which affect the capitalization or discount rate

14. It is generally accepted that the capitalization rate is equivalent to the discount rate less:

    a. Short-term growth rate
    b. Long-term sustainable growth rate
    c. Equity risk premium
    d. Risk free rate

15. Which variable below is NOT included in the Ibbotson Build-Up Method?

    a. Risk free rate of return
    b. Beta
    c. Size premium
    d. Specific company risk

16. Which component of the Ibbotson Build-Up Method relates to the "unsystematic risk" associated with a particular business entity?

    a. Risk free rate
    b. Equity risk premium
    c. Beta
    d. Specific company risk premium

17. Which of the following is NOT an assumption of the Capital Asset Pricing Model (CAPM)?

    a. Investors are risk averse
    b. There are no taxes and no transactional costs
    c. The rate received from lending money is the same as the cost of borrowing
    d. All investors do not have identical investment holding periods

18. Using the Modified Capital Asset Pricing Model a valuation analyst determines beta = 1.08. This means:

    a. The subject company is no more or no less volatile than the industry
    b. The subject company is less volatile than the industry
    c. The subject company is more volatile than the industry
    d. The subject company has no relative market risk

19. WACC can add versatility to the valuation, in that a valuation analyst could change the capital structure of an entity when valuing a non-controlling (i.e., minority) interest.

    a. True
    b. False

20. If a valuation analyst uses the weighted average cost of capital (WACC) and is valuing only the equity of the company, the valuation analyst would:

    a. Capitalize equity and ignore the debt
    b. Capitalize invested capital then subtract existing deb
    c. Determine the present value of the debt only
    d. Capitalize the cash flow net of debt

21. The criteria for companies included in the measurement data used to determine the equity risk premiums found in the Duff & Phelps Risk Premium report would include all EXCEPT:

    a. Must be publicly traded for 5 years
    b. Must have sales greater than $1 million in any of the previous 5 years
    c. Cannot be a financial service company
    d. EBITDA can either be negative or positive based on the most recent 5 year average

22. The Duff & Phelps equity risk premium measurements are sorted into _____ measures of size.

    a. five
    b. eight
    c. ten
    d. twelve

23. What component of cost of capital using a build-up method would the Duff & Phelps data help you determine?

    a. Company specific risk
    b. Equity risk premium
    c. Risk free rate
    d. Beta

24. What are the four general risk factor categories of the risk rate component model (RRCM)?
    a. Competition, financial strength, profitability and stability of earnings, and management ability and depth
    b. Competition, national economic effects, local economic effects, and depth of management
    c. Local economic effects, financial strength, market stability, and profitability and stability of earnings
    d. National and local economic effects, financial strength, management ability, and competition